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B.Sc. Part III (Maths.) (Hons.) 7th Paper

Diff. eqns (contd.)

Standard Form III

1. Solve $pz = 1+q^2$.

Soln. The given equation

$$pz = 1+q^2 \quad \text{———} (1)$$

It is of the form $f(p,q,z)=0$.

Put $u = x + ay$

$$\Rightarrow \frac{\partial u}{\partial x} = 1, \quad \frac{\partial u}{\partial y} = a$$

$$\therefore p = \frac{\partial z}{\partial x} = \frac{dz}{du}\cdot\frac{\partial u}{\partial x} = \frac{dz}{du}$$

$$q = \frac{\partial z}{\partial y} = \frac{dz}{du}\cdot\frac{\partial u}{\partial y} = a\frac{dz}{du}.$$

Putting the values of $p$ and $q$ in (1), we get

$$z\frac{dz}{du} = 1 + a^2\left(\frac{dz}{du}\right)^2$$

$$\Rightarrow a^2\left(\frac{dz}{du}\right)^2 - z\frac{dz}{du} + 1 = 0$$

$$\Rightarrow \frac{dz}{du} = \frac{z \pm \sqrt{z^2-4a^2}}{2a^2}$$

$$\Rightarrow \frac{dz}{z \pm \sqrt{z^2-4a^2}} = \frac{du}{2a^2}$$

$$\Rightarrow \frac{z \mp \sqrt{z^2-4a^2}}{\left(z \pm \sqrt{z^2-4a^2}\right)\left(z \mp \sqrt{z^2-4a^2}\right)} dz = \frac{du}{2a^2}$$

$$\Rightarrow \frac{z \pm \sqrt{z^2-4a^2}}{\cancel{z^2} - \cancel{z^2} + 4a^2} dz = \frac{du}{2a^2}$$

$$\Rightarrow \left(z \mp \sqrt{z^2-4a^2}\right) dz = 2du$$

Integrating, we get

$$\Rightarrow \int z\,dz \mp \int \sqrt{z^2-4a^2}\,dz = 2\int du$$

$$\Rightarrow \frac{z^2}{2} \mp \left[\frac{z}{2}\sqrt{z^2-4a^2} - \frac{4a^2}{2}\log\left(z+\sqrt{z^2-4a^2}\right)\right] = 2u + \frac{b}{2}$$

$$\Rightarrow z^2 \mp \left[z\sqrt{z^2-4a^2} - 4a^2\log\left(z+\sqrt{z^2-4a^2}\right)\right] = 4u+b$$

$$\Rightarrow z^2 \mp \left[z\sqrt{z^2-4a^2} - 4a^2\log\left(z+\sqrt{z^2-4a^2}\right)\right] = 4(x+ay)+b.$$

This is the complete integral.